Conflict and Cooperation within the Family, and between the State and the Family, in the Provision of Old-Age Security

Alessandro Cigno

No. 22
May 2014
Conflict and Cooperation within the Family, and between the State and the Family, in the Provision of Old-Age Security.*

Alessandro Cigno
University of Florence and CHILD

Abstract

The early contributions to the economic literature on this subject assume that only market goods yield utility, and that the only way adults can secure the consumption of these goods in old age is by saving. More recent contributions recognize that the elderly derive utility also from the care and attention they may receive from their relatives, and that they may eventually become too old or infirm to keep control over their own money. According to a branch of the literature, parents negotiate the provision of personal services directly with their children. According to another, such negotiations are made unnecessary by self-enforcing family rules. The present paper reviews the existing literature and addresses the question whether public provision of pensions and other facilities for the elderly raises welfare even when it crowds out family-level arrangements.

JEL: D7, D82, D91, H2, H31, H5, I2, J1.
Keywords: Private savings, public pensions, inter-vivos transfers, bequests, care of the elderly, family rules, fertility, longevity, informational asymmetries.

1 Introduction

This paper is about the different ways in which people, singularly or collectively, make provision for old age. The early contributions to this subject were based on the assumption that anything a person may want in old age can be bought from the market, and that all an adult can do

*This is an extensively revised version of the paper presented to the Workshop on the Economics of Population Ageing held at Harvard University on 26-27 September, 2013. Comments by John Piggott, Alan Woodward and other workshop participants are gratefully acknowledged.
to have money to spend in old age is save. More recently, the economic literature has come to terms with two facts of life. First, that the old attach importance not only to market goods, but also to the care and attention they receive from their near and dear. Second, that there may come a time when a person is too old or infirm to keep control over the use of her own savings or pension. Those who first wrote on the subject could not have been unaware of these facts, but probably took it for granted that the family, though never explicitly mentioned, would take care of all that. We had to wait for Becker (1974) and Bernheim et al. (1985) for these issues to be addressed within a formal model.

The literature on which we draw is largely theoretical, but we give also a summary account of that subset of the empirical literature which directly tests the predictions made by the different theories. To facilitate comparisons, the models will be shorn of inessential details, and re-exposed using a common notation. The life-cycle is divided into three phases or periods, labelled $p = 0, 1, 2$. A person is an infant in period 0, an adult in period 1, old in period 2. Adults can work and have children. Infants and the old can do neither. The choice of retirement date (i.e., in our framework, the possibility of choosing when period 1 ends and period 2 starts) is the subject of a distinct sub-literature, and will not be addressed here. Unlike the old, who may have accumulated savings or entitlements to a public pension, infants are entirely dependent on others, normally on their parents. This asymmetry is often ignored in the economic literature, especially in that specifically concerned with old age, where period 0 is typically left out of the picture.

Each adult is endowed with a fixed amount of time (normalized to unity) that has to be divided between paid work, care of others and leisure. Where inessential to the argument, leisure will be treated as a constant, and normalized to zero. To simplify the exposition, we will assume that the utility function is time-separable, and use period-specific felicity functions to allow for possible time-preference. For a good part of the paper, we will abstract from sex differentiation, and refer to the generic individual as "she". The complications associated with sexual reproduction, and with couple formation and dissolution, will be addressed where essential to the story.

In Section 2, we assume that individual decisions are subject only to the law of the land, and that the government’s only role is to enforce formal contracts. That role is much enhanced in Section 3, where we introduce a public pension system and enquire whether this form of government intervention enhances welfare. In Section 4, we bring in family rules, explain why they emerge and persist, and show how these rules

---

1 For a systematic exposition, see Fenge and Pestieau (2005).
and the number of persons complying with them are affected by public pension provision. Section 5, finally, examines the optimal (second-best) mix of public pensions, public services or facilities for the elderly, child benefits and public education, taking into account the effect that these policies will have on the viability of family-level arrangements.

2 Savings, gifts and care of relatives

In the models examined in this section, the government’s only role is to enforce contracts. In Subsection 2.1, individuals interact only through the market. In subsections 2.2 and 2.3, they interact also within the family, but such interactions are not the object of a legally enforceable contract. Prices, in particular the interest factor denoted by $r$, and the wage rate denoted by $w$, are taken as given.

2.1 Life-cycle theory

In the basic life-cycle model,² the individual derives utility from her own lifetime consumption stream only. Therefore, she will not make gifts or spend time doing anything other than paid work. Let $c_p$ and $s_p$ denote, respectively, consumption and savings in period $p$ of a person’s life. This person chooses $(c_0, c_1, c_2, s_0, s_1)$ to maximize her lifetime utility,

$$ U = u_0 (c_0) + u_1 (c_1) + u_2 (c_2), \tag{1} $$

where the function $u_p (.)$ is increasing and concave, subject to the period budget constraints

$$ c_0 + s_0 = 0, \tag{2} $$
$$ c_1 + s_1 = rs_0 + w \tag{3} $$

and

$$ c_2 = rs_1. \tag{4} $$

In the absence of other restrictions, this person will borrow in period 0 (choose $s_1 < 0$) and lend in period 1 (choose $s_1 > 0$) to the point where her marginal rate of substitution (MRS) of present for future consumption is the same in both periods and equal to the opportunity-cost of current consumption,

$$ \frac{u'_0 (c_0)}{u'_1 (c_1)} = \frac{u'_1 (c_1)}{u'_2 (c_2)} = r. $$

As everybody faces the same $r$, the MRS will be equalized across individuals as well as over the life cycle of each individual, and the allocation will then be efficient at the given prices.

²See Modigliani (1986).
Little of substance changes if we assume that this person will survive to period 2 with probability $\pi$, and that the maximand is consequently an expectation,

$$E(U) = u_0(c_0) + u_1(c_1) + \pi u_2(c_2), \ 0 < \pi < 1.$$  \hspace{1cm} (5)

Clearly, the higher is $\pi$, the less negative is $s_0$ and the more positive $s_1$. Savings increase with life expectancy ("longevity").

With or without uncertainty, this model overlooks two facts of life. One is that consumption in any period of life cannot fall below some positive subsistence level, denoted by $\xi$,

$$c_p \geq \xi.$$  \hspace{1cm} (6)

The other is that infants cannot sign a legally binding credit contract and, as a consequence, cannot borrow from the market,

$$s_0 \geq 0.$$  \hspace{1cm} (7)

With these additional restrictions, if an adult has children at all, she will set these their $c_0$ equal to zero, and none of them will live to be an adult. Before fretting about consumption in old age, therefore, we must worry about consumption during infancy. Indeed, as births are more easily prevented than procured, we must ask ourselves why people have children.

While still remaining within the life-cycle logic, Eckstein and Wolpin (1985) and several others in their wake deal with both these difficulties by assuming that the number of children a person has, denoted by $n$, yields direct utility. This is tantamount to saying that children are a consumption good like food or entertainment. The only difference is that the former are domestically produced with market inputs and the parent’s own time rather than bought ready-made from the market like the latter. Given that the children’s well-being does not yield utility, the parent will set her children’s period-0 consumption equal to $\xi$. Denoting by $c(n)$ the minimum amount of parental time that is needed to bring up $n$ children, the cost of children function is then

$$C(n, w) = \xi n + wc(n).$$  \hspace{1cm} (8)

In this extended version of the life-cycle model, an adult born at date $t$ chooses $(c_1^t, c_2^t, s^t, n^t)$, where $s^t$ is the amount she saves in period 1, to maximize

$$U^t = u_1(c_1^t) + u_2(c_2^t) + v(n^t),$$  \hspace{1cm} (9)
where $v(n^t)$ is the utility she derives from $n^t$, and $v(.)$ is increasing and concave, subject to the period budget constraints
\[ c_1^t + s^t + C(n^t, w) = w \] (10)
and
\[ c_2^t = rs^t, \] (11)
to the time constraint
\[ c(n^t) \leq 1 \] (12)
and to the physiological constraints
\[ 0 \leq n^t \leq \varphi, \] (13)
where $\varphi$ is the maximum number of children she can bear ("fecundity").

For an interior solution, the function $c(\cdot)$ will have to be either linear or convex (no economies of scale in child rearing). If that is the case, (12) – (13) will be slack, the benefit of an extra birth will be equated to the cost,
\[ \frac{v'(n^t)}{u_1'(w - s^t - C(n^t, w))} = \xi + wc'(n^t). \] (14)
and the MRS of adult for old-age consumption will be equated to the opportunity-cost of adult consumption,
\[ \frac{u_1'(w - s^t - C(n^t, w))}{u_2'(rs^t)} = r. \] (15)
If $n^t$ is chosen positive by (i.e., if the solution is interior for) at least some of the adults born at $t$, humanity will not vanish at date $t + 3$. Given that $c_0'$ is fixed at $\xi$, however, there is nothing to ensure that the MRS of infant for adult consumption will be equal to that of adult for old-age consumption. Therefore, the life-cycle allocation of consumption will generally be inefficient.

This model is generally referred to as the standard overlapping generations model (OGM). That terminology is uninformative, however, because overlapping generations are a feature of all models with a realistic demographic structure. What distinguishes this from other overlapping-generations models is that individuals are only concerned about their own lifetime consumption. For this reason, we prefer to label it the augmented life-cycle model.
2.2 Descending altruism

In a series of articles and ultimately in the Treatise, Gary Becker assumes that an adult regards not only the number ("quantity"), but also the well-being ("quality") of her children as goods (the so-called "descending altruism" assumption). The utility of a person born at date 1 may then be written as

\[ U^1 = u_0 (c^1_0) + u_1 (c^1_1) + u_2 (c^1_2) + v (n^1) n^1 U^2, \]  

where

\[ U^2 = u_0 (c^2_0) + u_1 (c^2_1) + u_2 (c^2_2) + v (n^2) n^2 U^3, \]

\[ U^3 = u_0 (c^3) + u_1 (c^3_1) + u_2 (c^3_2) + v (n^3) n^3 U^4 \]

and so on. This makes a person's utility a function of the number and lifetime consumption streams of all her descendants ("dynastic utility function"). In this version of the model, \( c^t_0 \) is bought ready-made from the market. In fuller versions, it is domestically produced with market inputs and parental time according to a concave production function. When this production function is substituted for \( c^t_0 \), \( u_0 \) becomes a function of market inputs and parental time ("care").

Let \( m^t \) and \( b^t \) denote, respectively, the amount of money (or, equivalently, market goods) that a person born at date \( t \) gives each of her children in period 1 and period 2 of her own life. A person born at date 1 will then choose \((n^t, m^t, b^t, s^t)\) for \( t = 1, 2, 3 \ldots \) so as to maximize (16) subject to (12) – (13),

\[ c^t_0 = m^{t-1}, \]  

\[ c^t_1 + s^t + n^t m^t + c (n^t) w = b^{t-1} + w \]  

and

\[ c^t_2 + n^t b^t = r s^t, \]  

taking \((m^0, b^0)\) as given. Becker interprets \( b^t \) as a bequest, but nothing of substance changes if we interpret it as an *inter vivos* transfer. On the face of it, this formulation allows for transfers to the old (\( b^t < 0 \)) as well as to infants (\( m^t > 0 \)), but we will see that this is illusory, and that personal savings remain the only source of old-age consumption.

At an interior solution, the benefit of having an extra child (the altruistic pleasure a parent receives from the contemplation of that child’s

---

3See, in particular, Becker and Barro (1988).
5In the more general version of the model mentioned in the last paragraph, the opportunity-cost of children figuring on the LHS of (18) will be larger than \( c (n^t) w \), because the parent will spend time also to produce \( m^t \).
happiness or utility) will be equated to the cost, which now includes the present value of the transfers the parent will make to the child,

\[
\frac{v'(n^t) U^{t+1}}{u'_1 (b^{t-1} + [1 - c(n^t)] w - n^t m^t - s^t)} = m^t + c'(n^t) + \frac{b^t}{r}.
\]

and the MRS of adult for old-age consumption to the opportunity-cost of adult consumption,

\[
\frac{u'_2 (r s^t - n^t b^t)}{u'_1 (b^{t-1} + [1 - c(n^t)] w - n^t m^t - s^t)} = r,
\]

for \(t = 1, 2, 3, \ldots\). For \(t = 2, 3, 4, \ldots\), that MRS will be equated also to the MRS of infant for adult consumption,

\[
\frac{u'_2 (r s^t - n^t b^t)}{u'_1 (b^{t-1} + [1 - c(n^t)] w - n^t m^t - s^t)} = \frac{u'_1 (b^t + [1 - c(n^{t+1})] w - n^{t+1} m^{t+1} - s^{t+1})}{u'_0 (m^t)}.
\]

In the absence of further restrictions, the allocation would thus be efficient at the given \(r\) and \(w\).

We have seen that, if (12) – (13) and (17) – (19) were the only constraints, dynastic utility maximization would bring efficiency. But there are further constraints. The fact that infants cannot borrow from the market need not be a problem in the present model, because they may receive transfers from their altruistic parent. As pointed out in Baland and Robinson (2002), the problem is rather that an infant cannot enter into a legally binding contract with her parent any more than she could with anyone else, and that a parent cannot leave a negative estate, because her heirs would not be obliged to accept it.\(^6\)

\[b^t \geq 0.\]  

As there is then no way to enforce repayment, there will be no loans from parents to children. This has two undesirable consequences. One is that infants will get no more than their parents are willing to give them as a present. The other is that the only way a parent can provide for her own old age is by lending to the market (saving) at the going interest rate, even if her children would be willing (but cannot commit) to pay her more. If (20) is binding, the MRS of present for future consumption will be larger for the children than for the parent, and the allocation will be inefficient.

Another potential source of inefficiency is identified in Becker (1974). Suppose that a child (a "rotten kid") takes pleasure in sabotaging a

\(^6\)There are few exceptions to this legal principle in the modern world.
sister’s career prospects. The parent can deter this kind of behaviour by making her transfers to each of her children a decreasing function of the recipient’s own income, because that will then make it in every child’s interest not to do anything that would make the other children’s income smaller. To make the deterrence last as long as possible, the parent should give her children money in the form of bequests rather than *inter vivos* transfers. On the other hand, however, *inter vivos* transfers relax the borrowing constraints that the recipient is more likely to face at the start than at the end of adult life, and will thus give her more utility than bequests.

Asymmetric information is yet another source of inefficiency. Suppose that a person’s earnings depend on time worked and luck, and that the parent observes her grown-up children’s realized earnings, but not the amount they work. If a child earns less than another, the parent cannot then tell if that is because the former was less lucky, or because she worked less hard than the latter. If work reduces utility, this will give rise to a moral hazard problem, and confront the altruistic parent with what Bruce and Waldman (1990) call the "Samaritan dilemma". On the one hand, an altruistic parent would in fact like to help a child in difficulty. On the other, however, she does not want to encourage shirking. The moral hazard problem is compounded by an adverse selection problem if "luck" consists in being born with high ability, and this ability is not directly observable by the parent. Cremer and Pestieau (1996) show that adverse selection is a further argument for giving children money in the form of bequests rather than *inter vivos* transfers, because that will give the parent more time to observe her children’s labour market performance and provide them with the right incentives.

Notice that none of these issues has any bearing on old-age provision. As mentioned at the start of this subsection however, in a more general version of the model, infants derive utility not only from market goods, but also from parental care, which does not have a perfect market substitute. The same may be said about the old. They, too, derive utility not only from the goods they can buy from the market, but also from the care they receive from their grown-up children, which does not have a perfect market substitute. That being the case, there is the problem of how to extract this care from reluctant adults (remember that, by assumption, the parent is altruistic towards her children, but the children are not altruistic towards their parent). One thing the parent can do is buy the care from her children. Given that the market does not provide a perfect substitute for this good, however, the children can form a cartel and use the ensuing monopoly power to extract the entire surplus generated from the transaction. More worryingly, the time may come
when the parent will be too old or disabled to exercise control over her own money, and the children can then take the money without giving anything in return.

The solution proposed by Bernheim et al. (1984) is that the parent should, in effect, organize an auction by pre-committing to leave her entire estate to the child who will give her the most care. For this to work, the testament should (a) be unamendable, and (b) include a clause stating that the money will go to someone other than the children if none of them supplies at least a certain specified amount of care. The authors argue that this all-or-nothing offer would give the parent the entire surplus generated by the exchange. There are two difficulties with this strategy however. One is that testaments are amendable. The other, first noted in Cigno (1991) and recalled in Cigno (2006b), is that the children can counter the parent’s move by drawing up a perfectly legal contract among themselves, in which they agree that one of them will give the parent the minimum amount of care needed to inherit the lot, keep enough of the inheritance to compensate her for the care given, and share the rest equally with her sisters.

The descending-altruism hypothesis underlying all these models is tested in Altonji et al. (1992, 1997) and found wanting. In the second of those articles in particular, the authors use micro-data to test whether a one dollar increase in the income of a parent who actually makes transfers to a child, accompanied by a reduction of the same size in the child’s income, would reduce the amount transferred by one dollar as the descending altruism hypothesis implies. Their finding that the transfer is reduced by only 13 cents rejects the hypothesis. Further evidence to the same effect will be examined in Section 4. Before going on to examine models based on different hypotheses, however, we want to pursue the point that, while concerned for her children’s well-being, an adult is also concerned for her own well-being, in particular for what will happen to her when she becomes old.

2.3 Ascending altruism

Nishimura and Zhang (1992) invert the direction of altruism by assuming that an adult derives utility from her parent’s old-age consumption ("ascending altruism"). For a person born at date $t$, therefore,

$$ U^t = u_0 (c_0^t) + u_1 (c_1^t) + u_2 (c_2^t) + \eta u_2 (c_2^{t-1}), \quad 0 < \eta < 1, \quad (21) $$

$^7$Descending and ascending altruism models are extreme cases of a more general model in which individuals derive direct utility from both their children’s and their parent’s consumption. As shown in Stark (1993), bilateral altruism does not guarantee efficiency unless a person values her parent’s and children’s consumption as much as her own.
where $\eta$ is a measure of filial piety. As in the augmented life-cycle model, this person regards anything she spends for her children as a cost, and will thus set $c_{t+1}$ at the lowest possible level, $\xi$. Here, however, the reason why she has children is not that they give direct utility, but rather that they may support her when she gets old. Therefore, children an investment, rather than a consumption good as in the augmented life-cycle and descending-altruism models. If the adult in question has sisters, and given that she derives utility from her parent’s old-age consumption, not from her own contribution to it, there is a potential free-riding problem. Nishimura and Zhang (1992) and several others in their wake assume, however, that the problem will not arise. We will start by assuming this, and then look at a more recent literature where that assumption is dropped.

As an adult cannot decide how much her children will give her when she gets old, the decision process is modelled as a non-cooperative Nash game.\(^8\) Let $p_t$ denote the amount that an adult born at date $t$ (and, for the no-free-riding assumption, each of her tween sisters) gives her parent as a present at date $t+1$. As the amount she herself consumed and the number of children her parent had at date $t$ are given constants, the adult in question will choose $(n_t, p_t, s_t)$ so as to maximize (21) subject to

\[
\begin{align*}
c_1^t + p_t + s_t + C (n_t, w) &= w, \\
c_2^t &= n_t p_{t+1} + rs^t
\end{align*}
\]

and

\[
c_{t-1}^2 = p_{t-1} n_{t-1} + rs_{t-1}^t,
\]

taking $p_{t+1}$ and $s_{t-1}$ as parameters.

In equilibrium, an adult will equate the marginal utility that she egoistically derives from her own current consumption to the marginal utility that she altruistically derives from her elderly parent’s current consumption,

\[
u'_1 (c_1^t) = n_t^{-1} \eta u'_2 (c_2^{t-1}),
\]

and her marginal valuation of her own current consumption in terms of her own future consumption to the opportunity-cost, represented by the marginal return to children,

\[
\frac{u'_1 (w - p_t - s^t - \xi n_t - wc(n_t))}{u'_2 (n_t p_{t+1} + rs^t)} = \frac{p_{t+1}^t}{\xi + wc'(n_t)}.
\]

For $n_t$ to be positive, this return must be at least equal to $r$,

\[
\frac{p_{t+1}^t}{\xi + wc'(n_t)} \geq r.
\]

\(^8\) The authors investigate also the possibility of a Stackelberg game.
If the marginal cost of children is increasing in $n^t$, this portfolio condition will be satisfied as an equation. In that case, the adult will save and have children. Otherwise, she will save nothing and rely entirely on her children for old-age support. Will the resulting allocation be efficient?

In steady state, an efficient allocation $(n, p, s)$ maximizes

$$U = u_1(c_1) + u_2(c_2) + \eta u_2(c_2)$$

subject to

$$c_1 + C(n, w) + p + s_1 = w$$

and

$$c_2 = s_1 r + nx.$$ 

Therefore, it satisfies

$$\frac{u_1'(w - p - s_1 + C(n, w)+)}{u_2'(c_2)} = (1 + \eta) \frac{p}{\xi + wc' (n)} \quad (24)$$

and

$$\frac{p}{\xi + wc' (n)} \geq r. \quad (25)$$

As the RHS of (24) is larger than that of (22), it is clear that the game will result in inefficiently low transfers to parents. Intuitively, that is because, in deciding how much to give her parent, an adult takes into account the altruistic pleasure she herself derives from this act, but not the egoistic pleasure that she would derive from receiving the same amount from each of her own children.

The no-free-riding assumption is dropped in Pezzin et al. (2007). The model has a two-stage game structure. The second stage determines how much each child will give the parent conditional on residential arrangement. The first stage determines the residential arrangement. Assuming that at most one of the children will live with the elderly parent at the second stage of the game, and that close proximity will make this child less prone to gamble on her sisters picking up the bill if she reduces her own contribution to the parent’s consumption, this will put the coresident child at either a bargaining or a strategic disadvantage (depending on whether that stage of the game is cooperative or non-cooperative) vis-à-vis her sisters. Stage-one decisions will thus be taken with an eye to the repercussions they will have at the second-stage game. The model is solved by backward induction. The exact specification of the first-stage game (whether the parent is a player or a spectator, and whether the players move sequentially or simultaneously) makes it more or less likely
that the equilibrium will be a Pareto optimum, but such an outcome is not guaranteed in any case.\footnote{These predictions are arrived at by analogy with existing models (including models designed to explain the transfer behaviour of non-custodial parents towards their children), rather than by formally analyzing a purpose-built model.}

The picture becomes more complicated if we bring in sexual reproduction, because we must then recognize that a child has not one but two biological parents, and more complicated still if we allow for the possibility that either or both biological parents will form a new union, because the child would then have also a step-parent.\footnote{For a survey of these issues, see Lundberg and Pollak (2007).} A complication of a different kind arises if one of the (biological or step) parents becomes incapacitated in old age before the other does. Pezzin et al. (2009) develop two models to show how the presence of children affects the amount of care that the able-bodied parent will provide for the disabled one. As women tend to marry older men, the parent who becomes incapacitated first is likely to be the man.\footnote{As the models are not formally stated, these predictions are to be taken as logical implications of a plausible set of assumptions.}

The first of these models adapts Oded Stark’s idea, that parents make transfers to grandparents in order to impress on the children that this is the right thing to do ("demonstration effect"),\footnote{See Stark (1995), Cox and Stark (2005).} to the case of an able-bodied mother providing care for her disabled husband in order to impress on the children that they should do the same for her when she in turn becomes disabled. There is an implicit assumption here that the children will not see through their parent’s ploy even though they are getting ready to play the same trick on the grand-children.

In the second model, by contrast, the able-bodied mother provides care for the disabled father because she fears that the children will otherwise punish her by denying her care when she in turn becomes disabled ("punishment effect"). The assumption here is either that the children have an innate sense of justice, or that they are guided by some kind of family rule (an economic explanation of where such a rule might come from will be given in Section 4).

In an empirical application, Pezzin et al. (2009) find that the presence of a child does indeed increase the probability that the able-bodied mother will provide care for the disabled father. In the case of a step-child, the size of the effect increases with the length of time that the child lived with the step-mother.
3 Social security

We now bring in public old-age provision ("social security"). We will not go into the normative arguments for such a form of government intervention. Suffice to say that, if reducing inequality within or between cohorts were the object, this could be pursued just as well and possibly at lower administrative cost by taxing and subsidizing adults rather than the old. If there is a justification for social security, it must be that the policy enhances efficiency.

In the present section, we will retain the assumption that the period-2 consequences of period-1 fertility, labour and saving decisions are certain and, as we are eschewing distributional issues, assume that individuals are differentiated only by their date of birth. To simplify the exposition, we will further assume that the service is delivered by a public agency (the "pension system"), but there is no compelling reason why this should be so. The essence of social security is that everybody (or every member of some specified categories) is obliged to participate in it, not that the government has the service is delivered by a public body.\footnote{On the subject, see Diamond (1977).}

Let $l_t$ denote the amount of time worked at date $t + 1$ by the representative member of the cohort born at date $t$. The pension system collects a contribution,

$$\Theta^t = \Theta (wl^t), \quad \Theta'(wl^t) > 0,$$

from this person at date $t + 1$, and delivers a benefit,

$$\Phi^t = \Phi (\Theta^t), \quad \Phi'(\Theta^t) \geq 0,$$

at date $t + 2$. If $\Phi'(\Theta^t)$ is positive, we say that the system is Bismarckian.\footnote{After German Chancellor Otto von Bismarck (1815 – 1898).} If it is zero, we say that the system is Beveridgean.\footnote{After British scholar William Beveridge (1879 – 1963).} In the first case, $\Theta$ is forced savings. In the second, it is a pay-roll tax in all but name.

The system is said to be actuarially fair if the capitalized value of the contributions made by a person up to the date of retirement equals the present value of the pension benefits she expects to receive from that date on. In our simple time-framework, without uncertainty, this means

$$\Phi^t = r\Theta^t.$$

The difference between the capitalized value of the contribution made and the present value of the benefit received,

$$\tau^t = r\Theta^t - \Phi^t,$$

\footnote{On the subject, see Diamond (1977).} \footnote{After German Chancellor Otto von Bismarck (1815 – 1898).} \footnote{After British scholar William Beveridge (1879 – 1963).}
is an implicit pension tax (positive, zero or negative). The implicit marginal tax on labour,

\[
\frac{d\tau_t}{dt} = w\Phi' (wl_t) - w\Phi' (\Theta_l) \Theta' (wl_t) = w \left[ r - \Phi' (\Theta_l) \right] \Theta' (wl_t),
\]

is equal to \( w\Theta' (wl_t) \) if the system is Beveridgean, smaller than that if the system is Bismarckian. Therefore, a Bismarckian system is always less distortionary than a Beveridgean one.\(^{16}\)

If the contributions collectively made by each cohort are invested, and their capitalized value used to pay benefits to the same cohort a period later, we say that the system is fully funded. If they are partly or totally used to pay benefits to the members of the previous cohort, we say that the system is either underfunded or unfunded (or "pay-as-you-go"). Existing pension systems are largely underfunded.\(^{17}\) That being the case, the system administrators (or the political authorities behind them) can choose to bestow an implicit pension subsidy, or infict an implicit pension tax, on an entire cohort. An extreme case of the former are the "inaugural gains" enjoyed by the first cohort of old-age pensioners when an unfunded pension system is first set up, or an initially fully-funded system is turned into an unfunded one. An extreme case of the latter is the extra cost incurred by the cohort caught in the transition from an unfunded to a funded pension system, whose members have to pay contributions sufficient to finance not only the previous cohort’s, but also their own pension benefits.\(^{18}\)

At date \( t + 2 \), when the cohort born at date \( t \) is old, an unfunded pension system’s current account satisfies the identity

\[
\Phi' = n^t \Theta^{t+1} + \delta^t,
\]  

where \( \delta^t \) is the deficit (positive, negative or zero) and \( n^t \) the number of contributors per pension beneficiary. In the reference literature, the pension system is generally assumed to be fixed in advance, by (26), but the individual benefit is left to be determined ex post by (30) and

\(^{16}\)The distortion caused by the implicit tax on labour is larger if the pension contribution makes the contributor credit constrained, but Cigno (2008) shows a Bismarckian system is less distortionary than a Beveridgean one in that case too. For evidence, see Disney (2004).

\(^{17}\)See Cigno and Werding (2007).

\(^{18}\)This will give rise to a welfare loss even if the additional cost is spread over all post-transition cohorts; see Breyer (1989), Fenge (1995) and Cigno and Werding (2007).
thus independent of the individual contribution. On the face of it, therefore, the system is Beveridgean. In the same literature, however, it is generally assumed that individuals hold rational expectations. As every contributor then "knows" the relationship between what she pays and what she will receive, the pension system is effectively Bismarckian.

An unfunded pension system is financially sustainable if

$$\sum_{t=0}^{t=\infty} \frac{N^t}{R^{t+1}} \delta^t = 0,$$

where

$$N^t = \prod_{i=0}^{t-1} n^i$$

and

$$R^{t+1} = \prod_{i=1}^{t+1} r^i.$$ 

It will raise welfare if

$$\sum_{t=0}^{t=\infty} \frac{N^t}{\delta^t} \left( U^t_p - U^t \right) > 0,$$

where $U^t_p$ and $U^t$ denote the utility level with and without the pension system of the representative member of the cohort born at date $t$, and $\frac{1}{\delta^t}$ is this person’s welfare weight.

In the reference literature, the question whether a pension system is (a) financially sustainable and (b) welfare-enhancing is generally addressed by demonstrating that, if the economy is disturbed from an initial steady state with $U^t = U$ for all $t$ by the introduction of an unfunded pension system satisfying (30) with $\delta^t = 0$ for all $t$, it will converge to a new steady state with $U^t_p = U^t_p$ for all $t$, and checking whether $U^t_p$ is larger or smaller than $U$. Notice that, for $\delta^t = 0$, (28) and (30) together imply

$$n^t \Theta^{t+1} = r \Theta^t.$$ 

In the new steady state, therefore, the pension system will be actuarially fair if and only,

$$n = r.$$ 

In the neoclassical growth literature, where utility is assumed to depend on consumption only, (33) is referred to as the "golden rule" of saving, because it identifies the saving rate that maximizes the steady-state level of per-capita consumption. In the presence of an unfunded pension
system, however, (33) does not have this optimizing property. If $n$ is greater (smaller) than $r$, the system will generate an implicit pension subsidy (tax). This is known as the Aaron condition.\footnote{See Aaron (1996).} Furthermore, if children give their parents direct utility as in some of the models outlined in Section 2, the consumption-maximizing path would not necessarily coincide with the utility-maximizing one.

In the following subsections, we will examine the consequences of introducing an unfunded pension system, first in a small open economy and then in a closed one.\footnote{Once again, the issues will be re-analyzed using a common framework, which does not entirely coincide with that of any of the original authors.} We will eschew the intermediate case where a country is open, but sufficiently large to affect interest and wage rates.\footnote{This case is analyzed in an international general equilibrium setting by Börsch-Supan et al. (2006), and Attanasio et al. (2007). As shown in the latter, the results are qualitatively not very different from those of the closed economy case.} As in the existing literature, we will assume that the representative individual behaves in the way described by one or other of the models set out in Section 2. Again as in the existing literature, we will further assume (a) that the representative individual’s optimization problem has an interior solution (in particular, that the nonnegativity constraints on savings and bequests are always slack) and (b) that the representative individual internalizes the system’s financial requirements. As two of the most obvious sources of inefficiency, credit rationing (either in the market or within the family) and free-riding,\footnote{On the former, see Cigno (2008). On the latter, see Cigno (1991).} are thus swept under the carpet, the task of demonstrating a need for an unfunded pension system is all the more difficult.

In the closed-economy case, the picture is completed by adding a representative firm and letting prices be determined by market-clearing conditions.\footnote{This will prevent us from examining the consequences of market imperfections. For a general equilibrium analysis of labour market rigidities, see de la Croix et al. (2013).} Some of the contributions on which we draw assume technical progress, but others do not. To ensure comparability, and given that productivity growth is not our primary concern, we will then postulate an unchanging constant-returns-to-scale technology. To make the model analytically tractable, the literature in question resorts to specific functional forms.\footnote{Specific functional forms are not needed to get results in the small-open-economy case, but we will use them all the same to facilitate comparison with the closed-economy case.} In particular, it is usually postulated that the
contribution rate is constant,

$$\Theta^t = \theta w^t, \ 0 < \theta < 1,$$

and that the cost-of-children function for an adult born at \( t \) is of the form\(^{25}\)

$$C (n^t, w) = (1 - \theta) w \psi (n^t)^\mu, \ 0 < \psi < 1, \ \mu \geq 1, \quad (34)$$

so that

$$l^t = 1 - \psi (n^t)^\mu, \quad (35)$$

$$\Theta^t = \theta w [1 - \psi (n^t)^\mu] \quad (36)$$

and

$$\Phi^t = n^t (1 - \psi (n^{t+1})^\mu) \theta w + \delta^t. \quad (37)$$

For the no-free-riding assumption, (37) can be substituted into the representative adult’s period-2 budget constraint. Without loss of generality, the amount of money or goods that an adult born at \( t \) transfers to her parent at date \( t + 1 \) will be expressed as a share of the former’s full income,

$$p^t = q^t (1 - \theta) w, \ 0 \leq q^t \leq 1.$$

### 3.1 Pensions in the life-cycle model

The effects of unfunded social security have been studied, theoretically and empirically by, among others, Auerbach and Kotlikoff (1987), Rios-Rull (1996), Börsch-Supan et al. (2006), and de la Croix et al. (2013), using the basic version (i.e., with exogenous fertility) of the life-cycle model. Another strand of literature, including among others. Fanti and Gori (2013), Miyazaki (2013) and Cipriani (2014), conducts the same kind of exercise using the fertility-augmented version of the same model. For the reasons set out in Subsection 2.1, we will concentrate on the latter. Here, the representative member of the cohort born at date \( t \) maximizes a log-linear version of (9),

$$U^t = \ln c_1^t + \beta \ln c_2^t + \gamma \ln n^t, \ 0 < \beta < 1, \ 0 < \gamma < 1. \quad (38)$$

The marginal cost of children is assumed constant \( (\mu = 1)\).\(^{26}\) With the policy, the budget constraints are

$$c_1^t + s^t + \psi (1 - \theta) wn^t = (1 - \theta) w$$

\(^{25}\)The out-of-pocket component of the cost of a child is assumed proportional to the parent’s net full income, and subsumed in \( \psi (1 - \theta) w \). This suggests that the minimum a parent must spend for each child is not literally the market value of the survival rations, but a socially determined parameter reflecting the parent’s economic status.

\(^{26}\)Given that the number of children is as a good with diminishing marginal utility, we do not need an increasing marginal cost to get an interior solution.
\[ c_2^t = rs^t + n^t \left[ (1 - \psi n^{t+1}) \theta w \right] + \delta^t. \]

Differentiating the FOCs and solving by Cramer, we find

\[ \frac{ ds^t }{ d\theta } = -\frac{ \beta (1 - \psi n^t) rwA + BC }{ H } w < 0 \text{ for } \theta \text{ small}, \]

\[ \frac{ ds^t }{ d\delta^t } = -\frac{ A }{ H } w < 0 \text{ for } \theta \text{ small}, \]

\[ \frac{ dn^t }{ d\theta } = \frac{ \gamma r (1 - \psi n^t) + (1 + \beta) [\psi r + (1 - \psi n^{t+1}) n^t] n^t }{ H } w > 0 \]
and

\[ \frac{ dn^t }{ d\delta^t } = -\frac{ \gamma }{ H } r < 0, \]

where

\[ A = (1 - \gamma) \psi (1 - \theta) r - \theta (1 - \psi n^{t+1}) > 0 \text{ for } \theta \text{ small}, \]

\[ B = (1 - \psi n^{t+1}) \theta + \beta \psi (1 - \theta) r > 0, \]

\[ C = (1 - \psi n^{t+1}) \gamma r + n^t > 0 \]
and

\[ H = [(1 + \beta) A + \gamma B] rw > 0. \]

Therefore, the contribution rate raises the demand for children and reduces savings. The pension deficit also reduces savings, but has no effect on the demand for children. The intuitive explanation is that both \( \theta \) and \( \delta^t \) raise an adult’s marginal valuation of current in terms of future consumption, but only \( \theta \) affects the marginal cost of children.

In a small open economy, where interest and wage rates are exogenously determined, steady-state fertility will then be higher with than without social security. If \( n \) was at least as high as \( r \) (e.g., if the economy was travelling along a golden-rule path) without social security, it will then be higher with it. If that is the case, the policy will generate an implicit pension subsidy and unambiguously raise steady-state utility. Otherwise, the policy will generate an implicit pension tax and thus possibly (not certainly, because \( n \) will still rise) reduce steady-state utility. These results are summarized in Table 1.

Does the same apply to a closed economy? Let \( Y^t, L^t \) and \( K^t \) denote, respectively, output, labour and capital per firm at date \( t \). Let the production function be Cobb-Douglas,

\[ Y^t = (K^t)^\alpha (L^t)^{1-\alpha}, \quad 0 < \alpha < 1, \]
so that the representative firm’s profit at date $t$ is

$$\Pi^t = (K^t)^\alpha (L^t)^{1-\alpha} - r^t K^t - w^t L^t. \quad (39)$$

Using the representative individual’s FOCs for the maximization of (38), and the representative firm’s FOCs for the maximization of (39), and assuming that capital vanishes after one period, Miyazaki (2013) shows that the equilibrium sequence converges in one period to a unique steady state characterized by positive income per adult, $y \equiv \frac{Y}{N}$, and positive fertility rate, $n$. Both $y$ and $n$ are increasing in $\theta$. Therefore, the pension system is viable, and raises steady-state utility.\footnote{If the monetary cost of children is independent of $(1-\theta) w$, however, the effects of $\theta$ can be signed only if capital is unimportant to production ($\alpha$ close to zero), in which case $n$ and $U$ are increasing in $\theta$ only up to a certain point, and then decreasing.}

These results are summarized in Table 1.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
& Fertility & Savings & Transfer to child & Transfer to parent & Utility \\
\hline
\(\theta\) & $> 0$ & $< 0$ & $0$ & $0$ & $> 0$ \\
\(\delta\) & $0$ & $< 0$ & $0$ & $0$ & $> 0$ \\
\hline
\end{tabular}
\caption{Pension policy effects in the augmented life-cycle model}
\end{table}

\begin{itemize}
\item $\theta =$ pension contribution rate (steady-state effects)
\item $\delta =$ deficit-financed share of pension benefits (off-steady-state effects)
\end{itemize}

The effects of an exogenous increase in longevity are examined in Cipriani (2014), where individuals survive to period 2 with probability $\pi$. In steady state, a higher $\pi$ is associated with a lower $n$ and a higher $y$. For any positive $\theta$, a higher $\pi$ is associated also with a larger $\Phi$ if $\alpha < 1/2$, with a smaller one if $\alpha \geq 1/2$. With or without survival uncertainty, therefore, the augmented life-cycle model predicts that social security raises fertility. With survival uncertainty, it predicts that increasing longevity does not imperil an unfunded pension system’s sustainability.

This appears to be at odds with the experience of the countries that have given themselves a universal public pension system. Since the end of World War II, life expectancy has risen as much in these countries as anywhere else, but completed fertility has fallen more than elsewhere. To counter the consequent increase in the age-dependence ratio, many of these countries have adopted a range of pro-natalist policies, including fertility-related pension benefits. In France, Germany and Sweden, for example, a woman is credited with a notional pension contribution for each child she has.\footnote{See Cigno and Werding (2007).}
The macroeconomic consequences of linking individual pension benefits to individual fertility decisions are examined in Fanti and Gori (2013). Here, the benefit formula is

$$\Phi_i = \theta [\omega n_i + (1 - \omega) n] w, \ 0 < \omega < 1,$$

where $\Phi_i$ denotes the pension benefit and $n_i$ the number of children of individual $i$. This formula rewards (penalizes) individuals with more (less) than the average number of children, denoted by $n$. The authors consider not only the case where individuals hold rational expectations, but also that in which they hold static ones. In the former, the model economy converges monotonically to a steady state with constant $n$ and $y$. In this long-term equilibrium, $n$ is increasing, and $y$ decreasing, in $\omega$. The policy thus achieves the aim of raising the aggregate fertility rate at the price of a reduction in per-capita income.

The static-expectations case is more problematic. If the value of $\alpha$ is at or above a certain level, and whatever the other parameter values, the economy will converge monotonically to a new steady state with positive $n$ and $y$. For lower values of $\alpha$, and depending on the other parameter values, the equilibrium sequence may follow an oscillatory trajectory either convergent to, or divergent from a new steady state. Therefore, linking individual benefits to individual fertility decisions may destabilize the economy.

### 3.2 Pensions in the presence of descending altruism

The policy analysis of the descending-altruism model set out in Subsection 2.2 is very complex, because a change in either $\theta$ or $\delta^t$ triggers substitutions not only of $c_1^t$ for $c_2^t$, and $n^t$ for $c_1^t$, as in the augmented life-cycle model, but also of $mt^t$ for $b^t$. In fuller versions of the descending-altruism model, where $c_0$ is domestically produced with money and parental time, there is also a substitution of money for the parental time. We will then examine the effects of pension policy in the simplified model used by Becker and Barro (1988), and Barro and Becker (1989).\(^\text{29}\)

The utility function is a special case of (16),

$$U^t = (c^t)^\sigma + (n^t)^{-\epsilon} n^t U^{t+1}, \ 0 < \sigma < 1, \ 0 < \epsilon < 1, \ \sigma + \epsilon < 1. \quad (40)$$

Here, $c^t$ denotes post-infancy (adulthood plus old-age) consumption for a person born at date $t$. In contrast with the more general version of the model, therefore, the only way this person can raise a child’s utility (“quality”) is by increasing $b^t$. The marginal cost of children is again

\(^{29}\text{Much of what follows is not in those articles, where the effects of pension policy are discussed without formal analysis.}\)
constant ($\mu = 1$). As adulthood and old age are telescoped into one period, savings coincide with bequests,\(^{30}\)

$$s^t = b^t n^t.$$  

(41)

Without social security, this model would then be irrelevant to our subject matter because it would tell us nothing about the way individuals go about providing for their own old age. It is relevant in the presence of a unfunded pension system, however, because it tells us whether the system is sustainable, and will raise or lower welfare.

A person born at date 0 now maximizes the dynastic utility function

$$U^0 = \sum_{t=1}^{\infty} \left( N^t \right)^{1-\epsilon} \left( c^t \right)^{\sigma}$$

obtained by substituting

$$U^{t+2} = \left( c^{t+2} \right)^{\sigma} + \left( n^{t+2} \right)^{-\epsilon} n^{t+2} U^{t+3}$$

into

$$U^{t+1} = \left( c^{t+1} \right)^{\sigma} + \left( n^{t+1} \right)^{-\epsilon} n^{t+1} U^{t+2},$$  

(42)

for each cohort $t$, and to the pension system’s sustainability condition (31). Given the inter-cohort nature of this optimization, it is not possible to talk of the effects of pension policy on any single cohort without also talking of its effects on all other cohorts.

From the FOCs, we find

$$\frac{\left[ b^{t-1} + (1 - \psi n^t) (1 - \theta) w + \frac{n^t}{r} (1 - \psi n^{t+1}) \theta w + \frac{\delta^t - b^t n^t}{r} \right]^{\sigma-1}}{(n^t)^{-\epsilon} \left[ b^t + (1 - \psi n^{t+1}) (1 - \theta) w + \frac{n^{t+1}}{r} (1 - \psi n^{t+2}) \theta w + \frac{\delta^{t+1} - b^{t+1} n^{t+1}}{r} \right]^{\sigma-1}} = r$$

(44)

and

$$\frac{(1 - \epsilon) \left( n^t \right)^{-\epsilon} \left( c^{t+1} \right)^{\sigma}}{\sigma \left( c^t \right)^{\sigma-1}} = (1 - \theta) w \psi + \frac{b^t - (1 - \psi n^{t+1}) \theta w}{r}$$  

(45)

\(^{30}\)Life-cycle savings, not visible, are assumed to be "small".
The first of these conditions equates the MRS of $c_t$ for $c_{t+1}$ to the opportunity-cost of $c_t$. The second equates the marginal benefit of $n_t$, decreasing in the argument, to the marginal cost, decreasing in $\theta$ and increasing in $b_t$.

In steady state, (44) becomes

$$n = r^{\frac{1}{2}}$$  \hspace{1cm} (46)

This tells us that $n$ is independent of $\theta$. Given that the marginal cost of $n_t$ is decreasing in $\theta$ and increasing in $b_t$ for every $t$, however, $n$ can stay put as $\theta$ varies only if $b$ varies in the same direction as $\theta$. Therefore, $b$ is increasing in $\theta$. Notice that $n$ is larger than $r$. With social security, there will then be an implicit pension subsidy. The policy raises the steady-state utility level $U$.

We have seen that a government can raise utility in the long run by introducing an unfunded pension system. But, a government can also raise utility for current adults (its electors) at the expense of future ones. Suppose that $\delta^t$ is raised and some or all of $\delta^{t+1}, \delta^{t+2}, \delta^{t+2} \ldots$ reduced in such a way that (31) continues to hold. If $b^t$ and $n^t$ remained the same as $\delta^t$ increased, the LHS of (44) would become larger than the RHS. As this cannot be, $b^t$ or $n^t$ must then vary. Notice that the LHS of (44) is decreasing in $b^t$ and increasing in $n^t$. If fertility were exogenous, $b^t$ would rise until (44) were again satisfied. Given that fertility is endogenous, however, and that $b^t$ is a component of the marginal cost of $n^t$, $n^t$ will fall, and $b^t$ will rise more that it would if $n^t$ were exogenous (in other words, the parent will have fewer children and over-compensate each of the them for the reduced pension benefit). The effect on $s^t$ is ambiguous. These results are summarized in Table 2.

### Table 2. Pension policy effects in the descending-altruism model

<table>
<thead>
<tr>
<th>Fertility</th>
<th>Savings</th>
<th>Transfer to child</th>
<th>Transfer to parent</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0</td>
<td>$&gt; 0$</td>
<td>0</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$&lt; 0$</td>
<td>?</td>
<td>$&gt; 0$</td>
<td>0</td>
</tr>
</tbody>
</table>

$\theta$ = pension contribution rate (steady-state effects)

$\delta$ = deficit-financed share of pension benefits (off-steady-state effects)

In a closed economy,\textsuperscript{32} the effect of $\theta$ on $s$ will be moderated by endogenous changes in $r$ and $w$. Given that (46) must still hold, however, social security will still raise steady-state utility.

\textsuperscript{31}Without social security, therefore, $s$ would be below the golden-rule level. Hardly surprising in view of our early remark that, if children yield direct utility, the golden rule does not maximize steady-state utility.

\textsuperscript{32}In Barro and Becker (1989), $y$ grows at the constant rate $g$ because the authors
3.3 Pensions in the presence of ascending altruism

Given the additional complexities associated with social security, we will examine the effects of pension policy in the basic version only of the ascending-altruism model set out in Subsection 2.3. In Nishimura and Zhang (1992), Zhang and Zhang (1995), and Zhang (1995), the utility function is a log-linear version of (21),

\[ U^t = \ln c^t_1 + \beta \ln c^t_2 + \eta \ln c^t_2^{-1}, \quad 0 < \beta < 1, \quad 0 < \eta < 1. \]  

(47)

The marginal cost of children is increasing \((\mu > 1)\) in \(n^t\). The amount of money or goods that an adult born at \(t\) transfers to her elderly parent is expressed as a share of the former’s full income,

\[ p^t = q^t (1 - \theta) w, \quad 0 \leq q^t \leq 1. \]

Unlike \(\psi\), \(q^t\) is a choice variable. The budget constraints are now

\[ c^t_1 + s^t + [\psi (n^t) + q^t] (1 - \theta) w = (1 - \theta) w, \]  

(48)

\[ c^t_2 = r s^t + n^t [(1 - \theta) q^{t+1} + \theta (1 - \psi (n^{t+1}))] w + \delta^t \]  

(49)

and

\[ c_2^{t-1} = r s^{t-1} + n^{t-1} \{ (1 - \theta) q^t + \theta \left[ 1 - \psi (n^t) \right] \} w + \delta^{t-1}. \]  

(50)

At an interior solution, \((n^t, q^t, s^t)\) satisfy the FOCs

\[ (1 - \theta) q^{t+1} + \theta \left[ 1 - \psi (n^{t+1}) \right] = r \psi \mu (n^t)^{\mu-1}, \]  

(51)

\[ r s^{t-1} + n^{t-1} \{ (1 - \theta) q^t + \theta \left[ 1 - \psi (n^t) \right] \} w + \delta^{t-1} \]  

(52)

\[ = n^{t-1} \{ (1 - \theta) w - s^t - [\psi (n^t)^{\mu} + q^t] (1 - \theta) w \}, \]  

(53)

and assume exogenous technical progress of the Harrod-neutral variety at that rate. To make the results comparable with those of the models examined in the last subsection, we have set \(g = 0\). The production function displays constant returns to scale, but is not necessarily Cobb-Douglas as in the last sub-section.

33 Wigger (1999) adds a term increasing in \(n^t\), and thus introduces a demand for children as a consumption good in addition to a demand for children as an asset. To keep the two mechanisms separate, we do not follow him in this.

34 Were it not so, the solution would be at a corner, with fertility equal either to zero or to the lower of the maximum number of children that can be financed on credit and the physiological maximum.

35 We are ruling out the possibility, considered in Section 2.3, that the marginal return to children could be larger than \(r\), and \(s^t\) consequently equal to zero.
In steady state, these conditions become

\[(1 - \theta) q + \theta (1 - \psi n^\mu) = r\psi n n^{-1}, \quad (56)\]

\[rs + nw [(1 - \theta) q + \theta (1 - \psi n^\mu)] = \eta n [(1 - \theta) (1 - \psi n^\mu - q) w - s] \quad (57)\]

and

\[rs + nw [(1 - \theta) q + \theta (1 - \psi n^\mu)] = r\beta [(1 - \psi n^\mu - q) (1 - \theta) w - s]. \quad (58)\]

Dividing (57) by (58), we find

\[n = \frac{\beta}{\eta} r. \quad (59)\]

As in the descending-altruism model, therefore, \(n\) is independent of \(\theta\). Assuming that people care about their own old-age consumption at least a little more than they care about that of their parents \((\beta > \eta)\), \(n\) will again be larger than \(r\). Therefore, social security raises steady-state utility.

On the other hand, in view of (56),

\[
\frac{(1 - \theta) q + \theta (1 - \psi n^\mu)}{\psi n n^{-1}} = \frac{\eta n}{\beta n}
\]

As \(n\) is independent of \(\theta\), this equation makes \(q\) a decreasing function of \(\theta\). With \(n\) and \(q\) so determined, (57) tells us that \(s\) is increasing in \(\theta\). In steady state, therefore, social security has no effect on fertility, but raises savings and lowers filial support for elderly parents.

As we did in relation with the descending-altruism model, we can establish the effects of an increase in \(\delta^t\) accompanied by reductions in \(\delta^{t+1}, \delta^{t+2} \ldots\) such that (31) continues to hold. Differentiating (51) - (54) totally and solving by Cramer, we find

\[
\frac{dn^t}{d\delta^t} = 0,
\]

\[
\frac{dq^t}{d\delta^t} = \frac{\eta n^{t-1} (1 - \mu) r \psi \mu (n^t)^{\mu - 2}}{H} > 0
\]
and
\[
\frac{ds^t}{dt}\mathcal{H} = -\frac{(1 + \eta)(1 - \mu) r\psi\mu(n^t)^{\mu-2}n^t-1(1 - \theta)w}{H} < 0,
\]
where \( H \) is the Hessian determinant, positive at a maximum. This tells us that an adult receiving a windfall from the pension administration will donate part of this windfall to her elderly parent, and consume the rest. As the return to children is not affected, her fertility behaviour will not change, but her utility will rise. These results are summarized in Table 3.

| Table 3. Pension policy effects in the ascending-altruism model |
|---------------------------------|----------------|----------------|----------------|----------------|----------------|
| Fertility | Savings | Transfer to child | Transfer to parent | Utility |
| \( \theta \) | 0 | > 0 | 0 | 0 | \( \geq 0 \) |
| \( \delta \) | 0 | < 0 | 0 | < 0 | \( > 0 \) |

\( \theta = \) pension contribution rate (steady-state effects)
\( \delta = \) deficit-financed share of pension benefits (off-steady-state effects)

In a closed economy, the effect of \( \theta \) on \( s \) will again be moderated by endogenous changes in \( r \) and \( w \). Given that (59) still holds, however, \( n \) will again be larger than \( r \), and social security will again raise steady-state utility.

4 Family rules

In reality, individual actions are constrained not only by the law of the land, but also by cultural, familial or religious norms. In the economics literature, such norms are usually ignored or taken as given (as, for example, in the "punishment" model examined at the end of subsection 2.3).

An early analysis of the consequences these norms can have is in Neher (1971). The author models a primitive economy were property

\[36\text{The analysis is in Barro and Becker (1989). There, the production function (not necessarily Cobb-Douglas) displays constant returns to scale, and } y \text{ grows at the constant Harrod-neutral technical progress rate } g. \text{ To make the results comparable with those of the models examined in the last subsection, we assume } g = 0.\]

\[37\text{Wigger (1999) finds a non-linear relationship between } \theta \text{ on the one hand, and } n \text{ and } U \text{ on the other. Given, however, that he assumes endogenous technical progress, and that the model is a hybrid of the ascending-altruism and the augmented life-cycle model because children yield direct utility, his results are not easily comparable with those of the basic ascending-altruism model examined here. Besides, he assumes that } p \text{ is proportional to } w \text{ rather than } (1 - \theta)w, \text{ and that pension contributions are levied on full rather than actual income. These assumptions are hard to justify and do not allow the small-open-economy effects of } \theta \text{ to be signed.}\]
rights are vested in families, rather than individuals, and family income is distributed according to a "... share alike ethic whereby all members of the family have equal claim to the product whether they work or not." This ethic reduces the individual incentive to work hard and refrain from immediate consumption, because the benefit will have to be shared with other family members, and increases the incentive to have children, because the cost of raising them will be similarly shared. Production, consumption and asset accumulation will consequently be inefficiently low, and fertility inefficiently high. Arguably, norms like the one envisaged by Philip Neher have lost currency because they make everyone poorer. If a norm persists and is obeyed without or even against the enforcement apparatus of the modern state, it must be that it helps to solve a coordination problem, and is thus seen by everyone concerned as beneficial.

Beginning with Shubick (1981), there have been several attempts, including Kotlikoff et al. (1986), Kotlikoff (1988), Esteban and Sakovics (1993), Cigno (1993, 2006), Caillaud and Cohen (2000), Rangel (2000, 2003), Guttman (2001), Anderberg and Balestrino (2003), Lindbeck et al. (2003) and Lindbeck and Nyberg (2006), at explaining the emergence and persistence of social or family norms as the outcome of some kind of intergenerational game.

Rangel (2000, 2003) models the provision of what he calls "backward investment goods" (BACs) and "forward investment goods" (FIGs) as an infinitely repeated game between generations. As examples of the former, the author mentions unfunded social security and individual care of elderly parents. As examples of the latter, he gives parental or public investment in education, investment in infrastructure and the preservation of the environment. He shows that the provision of FIGs is sustainable as an equilibrium (not necessarily efficient) because these goods generate a surplus. BIGs, by contrast, do not generate a surplus, and their provision is sustainable as an equilibrium only if combined with the provision of FIGs.

Cigno (1993, 2006) reaches analogous conclusions by a different route. The earlier of those two papers establishes conditions such that a set of family rules is self-enforcing in the sense that it is in every family member's interest to obey them and have them obeyed. The later one takes the analysis further by establishing conditions such that, once in

---

38 As already pointed out, the first example is somewhat misleading. The pension benefits received by a cohort of individuals who did not pay pension contributions ("inaugural gains") are indeed BIGs, but those received by subsequent cohorts are not. In steady state, the net benefit of participating in the system is the same for every cohort (or increasing at the technical progress rate if there is any).
place, a set of rules will not be renegotiated, and may thus be regarded as the family-level equivalent of the political constitution that restricts the legislative powers of successive parliaments at the national level.\footnote{See Buchanan (1987).} In the following subsections, we will look at this approach in more detail, and examine the way in which family-level arrangements stand up to the competition of publicly provided old-age security.

### 4.1 A family constitution

In its simplest form, a family constitution prescribes (a) the minimum amount of money or goods, $z \geq \xi$, that an adult must transfer to each of her children if she has any, and (b) the minimum amount of money or goods, $x \geq 0$, that she must transfer to her parent if the latter obeyed the constitution in her turn.\footnote{We can safely assume that $z$ will be at least as large as $\xi$, and $x$ at least as large as $\xi$ divided by the number of children the parent has.} This gives each adult a choice of two strategies: cooperate ("comply" with family rules), or defect ("go it alone" in the market). The utility function is (1) as in the basic life-cycle model.

For a go-it-alone, the cost of having $n$ children is $\xi n + wc(n)$. As her children, if she decided to have any, would give her nothing when she got old (because, even if they chose to comply, the family constitution would allow them to give their parent nothing), this person will then choose $n = 0$. Given that she is past infancy, and that $u_0(c_0)$ is consequently a by-gone, the pay-off of the go-it-alone strategy is

$$V(r, w) = \max_s u_1(w - s) + u_2(rs)$$

and her choice of $s$ will thus satisfy

$$\frac{u'_1(w - s)}{u'_2(rs)} = r.$$  \hfill (60)

For a complier, the cost of having $n$ children is $zn + wc(n)$, at least as high as if she chose to go it alone. On top of that, however, she must give her parent a fixed amount of money $x$. Unlike the go-it-alone strategy, therefore, the comply strategy has a fixed cost. Next period, if her children also comply, the complier will receive $x$ from each of her children. In equilibrium, the pay-off of the comply strategy is thus

$$V^*(r, w, x, z) = \max_{n^c, s} u_1(w[1 - c(n)] - s - x - zn) + u_2(rs + xn).$$
The associated choice of \((n, s)\) will equate the complier’s MRS of adult for old-age consumption to the marginal return from having children,

\[
\frac{u'_1(w[1-c(n)] - s - x - zn)}{u'_2(rs + xn)} = \frac{x}{z + wc'(n)},
\]

and satisfy the portfolio condition that the return in question must be at least as large as the return to savings,

\[
\frac{x}{z + wc'(n)} \geq r.
\]

The last condition differs from the analogous one we encountered in the ascending-altruism model only in that the numerator of the LHS term is fixed by family rules, rather than chosen by the player’s children as in that model. As in that model, this condition will be satisfied as an equation, and the optimization will have an interior solution, if \(c'(.\) is an increasing function.

Like the generic adult in the ascending-altruism model, a complier will have children to the point where her marginal valuation of current consumption equals the benefit of an extra child. If, at that point, the return to children is equal to \(r\), she will also save. Otherwise, she will save nothing and rely entirely on her children for old-age support. Note that the return to children may be equal to \(r\) for the marginal child, but will be greater than \(r\) for inframarginal ones. Therefore, children will always generate a surplus. For complying to be the winning strategy, this surplus must be at least equal to the fixed cost of complying, \(x\).\(^{41}\) If that is the case,

\[
V^*(r, w, x, z) \geq V(r, w)
\]

and the set of comply strategies is a sub-game perfect Nash equilibrium.\(^{42}\) Otherwise, go-it-alone will be the winning strategy, and the adult in question will behave as in the basic life-cycle model.

Given that an infinite number of \((x, z)\) pairs may satisfy (63), which will prevail? Cigno (2006a) adapts the renegotiation-proofness selection criterion developed by Bernheim and Ray (1989) and Maskin and Farrell (1989) for a repeated game where the players are always the same, to a game like the present one where the players change at each round (as a cohort reaches adulthood and another becomes old). Compared with

\(^{41}\)In Cigno (1993, 2006), the marginal cost of children is constant as in the models examined in the last section. For (63) to hold, the return to children, also constant, must then be strictly larger than \(r\). As a result, compliers do not save.

\(^{42}\)for a demonstration, see Cigno (1993).
all the other constitutions which also satisfy (63), a renegotiation-proof constitution has the additional property that it is not Pareto-dominated by any of them.

A constitution prescribing \((x, z)\) will then be renegotiation-proof if and only if it maximizes

\[
U(n, s, x, z) = u_0(z) + u_1(w[1 - c(n)] - s - x - zn) + u_2(rs + xn)
\]

subject to (63). The associated allocation will satisfy (61) – (62),

\[
\frac{u'_0(z)}{u'_1(w[1 - c(n)] - s - x - zn)} \geq \frac{u'_1(w[1 - c(n)] - s - x - zn)}{u'_2(rs + xn)}
\]

and

\[
\frac{u'_1(w[1 - c(n)] - s - x - zn)}{u'_2(rs + xn)} = \frac{x}{z + wc'(n)} = n
\]

If (63) is slack, (65) will be satisfied as an equation, and the allocation will be a Pareto-optimum. Otherwise, the allocation will be a constrained Pareto-optimum.

The analysis can be extended in a number of directions. Rosati (1996) shows that allowing for a child’s survival beyond infancy (or, more generally, for her future ability to support her elderly parent) to be uncertain may give rise to a precautionary demand for savings, but does not alter the model’s predictions in any substantive way. Cigno and Rosati (2000) let a person’s utility depend not only on money or market goods, but also on the personal services (“care”) this person receives from her parent during infancy, and from her children in old age. The constitution is re-worded so that it requires every adult to give each of her children a basket of personal services and market goods yielding at least the same utility as a sum of money \(z\), and her parent a basket of personal services and market goods yielding at least the same utility as a sum of money \(x\). By allowing adults to pick the cost-minimizing mix of money and personal services with which to satisfy the constitutional requirements, this extension reduces the cost of complying, and makes it more likely that a self-enforcing family constitution exists. Anderberg and Balestrino (2003) interpret \(z\) as education.

Cigno (2006a) shows that little of substance changes if we put the number and utility of children in the parent’s utility function (“descending altruism”), in which case the constraint that an adult must give each of her children at least \(z\) may be slack, but the constraint that she must give her parent at least \(x\) will be tighter than ever. With this extension, the family constitution model becomes directly comparable with Becker’s (see Subsection 2.2). Given that an adult has two strategies
(comply or go it alone) in the former, but only one (go it alone) in the latter, maximized utility will be at least as high in the present model as in Becker’s. Descending altruism, however, brings back the incentive and sibling rivalry problems encountered in Subsection 2.2. Chang and Luo (2014) argue that these problems will go away if the family constitution is re-worded to say that each adult must divide her wealth into two parts, one to be bequeathed and shared equally among her children, and the other to be transmitted to them while the parent is still alive in accordance with the amount of care each of them gives to the parent. The authors demonstrate that this constitution will generate the right incentives, and be self-enforcing. They also conjecture that it will discourage grown-up children from colluding to give their elderly parent as little care as possible. (the problem raised at the end of Subsection 2.2).

A more fundamental difficulty arises if we introduce sexual reproduction, because the actor is then the couple rather than the individual, and each couple has two sets of parents and two sets of family rules to contend with. In traditional societies, the problem is solved by one the partners (usually the woman, but in some cases the man) leaving the family of origin and joining ("marrying into") that of the other partner. In modern societies, however, each partner maintains (or does not maintain, as the case may be) her or his links with the family of origin. We are not aware of any attempt at modelling the constitutional arrangement that would be appropriate for such a situation.

Evidence supportive of the hypothesis that a share of the adult population is governed by something resembling a family constitution is found by a micro-econometric study of transfer behaviour in Italy. Using a Bank of Italy household survey that, for a particular year, records monetary transfers to friends or relatives (rather than from them as for all previous and subsequent years), and thus allows the probability and size of the transfer to be related to the donor’s characteristics, Cigno et al. (2006) find that, other things being equal, both the probability and the size of the transfer are higher if the donor is credit rationed than if she is not. This rejects the hypothesis that those transfers are either gifts ("altruistic motive") or the visible counterpart of an unrecorded personal service ("exchange motive"), because in either of those cases the probability and the size of a transfer would be positively related to

---

43 Typically, there are also compensatory payments, by the man to the woman’s family of origin ("bride-price") if the woman is seen as an asset, the other way round ("dowry") if she is seen as a liability.

44 The probability of being rationed is endogenized using appropriate instrumental variables.

the donor’s wealth and thus, controlling for assets and earnings, lower if the donor is credit rationed than if she is not. By contrast, the finding does not reject the hypothesis that the transfers are made in compliance with family rules.

Consistently with evidence in Crimmins and Ingegneri (1990), Davis et al. (1997) and Cigno and Rosati (2000), that very young children and elderly parents receive primarily personal services, Cigno et al. (2006) find that monetary transfers go primarily to non-core resident children.

4.2 Social security vs. family constitutions

The effects of social security are more complex in the constitution model, even in its basic version, than in any of the other models considered so far, because the policy affects individuals differently according to which strategy they choose, and may affect their choice of strategy. Leaving the latter on one side for the moment, we will start by assuming (a) that every adult complies with some family constitution, and (b) that the optimization problem determining the representative complier’s family rules, and associated fertility and saving behaviour, has an interior solution. Assumption (a) implies that the aggregate fertility rate and savings per adult coincide with the number of children and the savings of the representative complier (identified by the c superscript),

\[ n = n^c \]

and

\[ s = s^c. \]

Assumption (b) implies that all the inequality constraints, (63) included, are slack for everyone, and that all the existing family constitutions are consequently efficient.

The constitution \((x^c, z^c)\) complied with by the representative adult, and the associated choice of \(n^c\) and \(s^c\), found maximizing

\[ U (x^c, z^c, \theta, \delta) = u_0 (z^c) + u_1 ((1 - \theta) w [1 - c (n^c)] - s^c - x^c - zn^c) + u_2 (rs^c + x^c n^c + \theta w [1 - c (n^c)] n^c + \delta), \]

satisfy

\[ \frac{u'_0}{u'_1} = \frac{u'_1}{u'_2} = n^c \]

and

\[ n^c = \frac{x + \theta w [1 - c' (n^c)]}{z + (1 - \theta) wc' (n^c)} = r. \]  

(67)

Under present assumptions, therefore, the aggregate fertility rate would be the same with or without social security, and equal to the interest
factor. No matter whether \( r \) is exogenous or endogenous, the policy would then leave \( U \) unchanged.

The effects of an exogenous change in either \( r \) or \( w \) on \( n^c \), \( s^c \), \( x^c \) and \( z^c \), found differentiating the FOCs totally and solving by Cramer, are

\[
\frac{\partial s^c}{\partial \theta} = \frac{\partial s^c}{\partial \delta} = \frac{\partial x^c}{\partial \theta} = \frac{\partial x^c}{\partial \delta} = \frac{\partial z^c}{\partial \delta} = 0
\] (68)

and

\[
\frac{\partial z^c}{\partial \theta} = -\frac{r w [1 - c (r)] (u''_1 + r^2 u''_2)^2}{H} > 0,
\] (69)

where \( H \) is the Hessian determinant (negative at a maximum). The economic intuition is straightforward. With the return to children fixed at \( r \), the policy does not affect portfolio choice. On the other hand, as the return to children is increasing in \( \theta \) and \( x^c \), and decreasing in \( z^c \), \( x^c \) must fall relative to \( z^c \) for that return to remain constant as \( \theta \) rises. Changes in \( \delta \) do not affect the return to children and thus the choice of \( s^c \), \( x^c \) and \( z^c \) (they will only affect adult consumption).

Let us now relax the assumption that (63) is slack for everybody. Let there be a continuum of individuals differentiated by a parameter, \( \zeta \), which raises the marginal cost of children. There will be a threshold value of this parameter, \( \zeta^m \), such that the pay-off of the comply strategy equals the pay-off of the go-it-alone strategy. For all individuals characterized by a value of \( \zeta \) higher than \( \zeta^m \), (63) will not hold. These individuals will go it alone. All individuals characterized by a value of \( \zeta \) lower than or equal to \( \zeta^m \) will comply. For those with \( \zeta \) equal to \( \zeta^m \), (63) will be slack and the domestic resource allocation will be a Pareto-optimum. For those with \( \zeta \) greater than \( \zeta^m \), (63) will be tight, and the domestic resource allocation will be a constrained Pareto-optimum.

Diversity carries two implications. The first is that compliers with different \( \zeta \) may have different \( x \) and \( z \) (different constitutions). The second is that, as not everybody will follow the same strategy, we can no longer talk of a representative adult, but we can still talk of a representative go-it-aloner and a representative complier. Further assuming that at least some individuals go it alone (have \( \zeta \) greater than \( \zeta^m \)), the aggregate fertility rate will then be lower, and savings per adult higher than, respectively, the number of children and the savings of the representative complier,

\[ n < n^c, \]

and

\[ s > s^c. \]
With social security, the pay-off of the comply strategy for an individual with cost parameter \(\zeta\) (\(\zeta\) for short) is

\[
V^*(r, w, x^\zeta, z^\zeta, \theta, \delta, \zeta) = \max_{s^*} u_1 \left( (1 - \theta) w [1 - c(r)] - s^\zeta - x^\zeta - (\zeta + z^\zeta) r \right) + u_2 \left( s^\zeta r + x^\zeta r + n\theta w [1 - c(r)] + \delta \right).
\]

Having established (in the last subsection) that every complier has \(r\) children, we have substituted \(r\) for both \(n\). For that same individual, the pay-off of the go-it-alone strategy will be

\[
V(r, w, \theta, \delta) = \max_{s^*} u_1 \left( (1 - \theta) w - s^\zeta \right) + u_2 \left( s^\zeta r + n [1 - c(r)] \theta w + \delta \right).
\]

The effects of a small change in \(\theta\) on the pay-offs of the two strategies are

\[
V^*_\theta = -w [1 - c(r)] \left( 1 - \frac{n}{r} \right) u'_1 \left( (1 - \theta) w [1 - c(r)] - s^\zeta - x^\zeta - (\zeta + z^\zeta) r \right)
\]
and

\[
V_\theta = -w \left( 1 - \frac{n}{r} [1 - c(r)] \right) u'_1 \left( (1 - \theta) w - s^\zeta \right),
\]
both negative because \(n\) is smaller than \(r\). Those of a small change in \(\delta\) are

\[
V^*_\delta = u'_2 \left( s^\zeta r + x^\zeta r + n\theta w [1 - c(r)] + \delta \right)
\]
and

\[
V_\delta = u'_2 \left( s^\zeta r + n [1 - c(r)] \theta w + \delta \right),
\]
both positive.

For an individual with \(\zeta = \zeta^m, \frac{u'_1}{u'_2} = r\) in both strategies. Given that the comply strategy has a fixed cost \(x^m\) (absent in the go-it-alone strategy), however, \(u'_1\) will be higher, and \(u'_2\) lower, in the former than in the latter. For this person, therefore, \(V^*_\theta\) is more negative than \(V_\theta\), and \(V^*_\delta\) less positive than \(V_\delta\). If either \(\theta\) or \(\delta\) increases, marginal compliers (those with \(\zeta\) equal to, or a little smaller than, \(\zeta^m\)) will turn into go-it-aloners.\(^{46}\) Notice that, the higher is \(r\), the more strongly negative is the effect of \(\theta\) on \(V^*\) and thus on the share of compliers in the adult population.

If we compare two alternative steady states, one with a high \(\theta\) and the other with a low \(\theta\), we will then find that the share of compliers is lower in the former than in the latter. In the absence of other effects, and given that a person has fewer children, makes smaller transfers and saves more if she goes it alone than if she complies, \(s\) will then be higher, and

\(^{46}\)Put more formally, \(m\) is increasing in \(\theta\) and \(\delta\).
$n$, $x$ and $z$ lower at the high than at the low $\theta$. These are composition effects. But there are also other effects.

In view of (68) – (69), an inframarginal complier (one who would comply at either value of $\theta$) would in fact have the same number of children, save the same amount and give her parent as much at the high as at the low value of $\theta$, but would give each of her children more at the former than at the latter. Conversely, and recalling that a go-it-aloner behaves in the way described by the basic life-cycle model, an inframarginal go-it-aloner would have no children and make no transfers to anyone at the high as at the low $\theta$, but would save less at the former than at the latter. These within-strategy adjustments will tend to offset the composition effects on per-adult savings, $s$, and per-adult transfers to each every old person, $x$, and each infant, $z$.

As pointed out in the last section, it does not make sense to talk of steady-state effects of $\delta$ because, in steady state, the deficit must be equal to zero for the pension system to be financially viable. It does make sense, however, to talk of the off-steady-state effects of making the pension deficit positive for one cohort and negative for another. In the presence of such a policy, family constitutions would instruct each adult to make a larger transfer to each of her children (hence, to each future adult) if she belongs to a cohort blessed with a positive deficit, than if she belongs to a cohort cursed by a negative one.

As public inter-cohort transfers would thus be neutralized by private ones, the policy would have no composition effects. There would be within-strategy adjustments however. Each complier would in fact make a larger transfer to each of her children, and each go-it-aloner would save more, if the deficit were positive than if it were negative. Fertility, transfers to the old and utility would not be affected. These predictions are summarized in Table 4.

**Table 4. Pension policy effects in the presence of family constitutions**

<table>
<thead>
<tr>
<th></th>
<th>Fertility</th>
<th>Savings</th>
<th>Transfer to child</th>
<th>Transfer to parent</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>$&lt; 0$</td>
<td>$\geq 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0$</td>
<td>$&lt; 0$</td>
<td>$&gt; 0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

$\theta =$ pension contribution rate (steady-state effects)

$\delta =$ deficit-financed share of pension benefits (off-steady-state effects)

As usual, in a closed economy, the effects of pension policy will be moderated by endogenous price adjustments. So long as not everybody complies, however, $n$ will still be lower than $r$. Therefore, the introduction or expansion of an unfunded pension system will still reduce
steady-state utility, and there will still be no scope for increasing a co-
hort’s utility at the expense of one or more others.

4.3 Evidence

The effects of pension policy on the fertility rate, and on either the sav-
ing or the productivity growth rate, have been studied empirically using
cross-country data, single-country time-series or pooled cross-country
and time-series data by a number of authors including Hohm (1975),
Ehrlich and Zhong (1998), Cigno et al. (2003a), Zhang and Zhang
(2004), Puhakka and Viren (2006) and Galasso et al. (2009). The time-
series studies use either co-integration or error-adjustment methods to
distinguish between short and long-term effects. All assume, implicitly
or explicitly, that factor prices are exogenously determined (small open
economy assumption).

All the studies mentioned and others besides … nd a strongly negative
effect of public pension provision on the fertility rate. Under the further
assumption that (a) the population is entirely covered by the public
pension system, (b) the pension contribution rate is exogenous, and
(c) the pension benefit is endogenously determined by the requirement
that the pension system must break even every year, Ehrlich and Zhong
(1998) find also that the pension contribution rate a negative effect, and
Zhang and Zhang (2004) that it has a positive one, of on the productivity
growth rate.

Assumptions (a) and (c) are somewhat arbitrary. In some countries,
for example Italy, public pension coverage has increased gradually since
the end of World War II and become universal only in the 1980s. In oth-
ers, for example in the US, a large part of the population is still without
public pension coverage. All public pension systems show deficits in
some years and (occasionally) surpluses in others. In general, therefore,
the average pension contribution rate reflects not only the rate charged
to individual participants, but also the share of the adult population
subject to those charges, and the extent to which the average pension
benefit is deficit financed.

Controlling for the real pension deficit (surplus) per adult, the real
interest rate (10-year bond yield), real male and real female wage rates,
and real income per adult (because labour may be rationed, and some of
the income comes from property rather than labour), Cigno and Rosati
(1992, 1996) and Cigno et al. (2003a) estimate the steady-state and
transitional effects of the share of the adult population covered by the
public pension system ("pension coverage")47 on both the number of

47 This is described by the authors as the "extensive" measure of pension coverage.
births per woman of fertile age (the "Total Fertility Rate") and the savings rate in Germany, Italy, UK and US. Cigno et al. (2003a) re-do the exercise for Germany alone using a VAR (vector auto-regressive) model that lets the data determine which of the variables are endogenous, and which exogenous. Savings and fertility are found to be endogenous, and the policy variables exogenous in the long run. In the short run, the deficit responds to savings behaviour. The estimated long-run elasticities are reported in Table 5. Similar estimates, not reported here, are obtained by Cigno and Rosati (1997) for Japanese savings alone.

According to these estimates, the expansion of an unfunded pension system holding the pension deficit and individual contribution rates constant (hence, an increase in the average pension contribution rate) would reduce the fertility rate, and raise the savings rate. An increase in the pension deficit holding coverage and individual contribution rates constant would raise fertility a little or leave it unchanged, and reduce savings or leave them unchanged.

<table>
<thead>
<tr>
<th>Table 5. Estimated pension policy effects (elasticities)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Germany</strong></td>
</tr>
<tr>
<td>Coverage</td>
</tr>
<tr>
<td>Deficit</td>
</tr>
<tr>
<td>Interest rate</td>
</tr>
<tr>
<td>Female wage</td>
</tr>
<tr>
<td>Male wage</td>
</tr>
<tr>
<td>Income</td>
</tr>
<tr>
<td><strong>Source:</strong> Cigno et al. (2003a)</td>
</tr>
</tbody>
</table>

These empirical findings reject the augmented life-cycle model, which predicts a positive fertility effect and a negative savings effect for the pension contribution rate, together with zero fertility and negative savings effects for the pension deficit (see Table 1). They reject also the descending-altruism hypothesis, according to which the contribution rate does not have any effect on either savings or fertility, but a deficit would reduce fertility (see Table 2), and the ascending-altruism hypothesis.

An "intensive" measure, defined as real pension benefits per person over the age of 65, is also tried but usually performs less well than the extensive measure because of collinearity with the pension deficit.

Cigno and Rosati (1992) estimate that as much as 60 percent of the fall in Italy’s Total Fertility Rate between the end of World War II and the end of the 1980s is explained by the expansion of _Previdenza Sociale._
according to which neither coverage nor the deficit has any effect on fertility, but the former raises and the latter lowers savings. By contrast, they do not reject the hypothesis that a share of the adult population is governed by a family constitution and the rest behave as in the basic life-cycle model, and that this share is affected by pension policy, because the estimated sign pattern is the same as in Table 4.

There is some micro-data evidence too. For example, Nugent and Gillaspy (1983) find that public pension provision discourages fertility. The evidence concerning the effect of public pension provision on individual savings is extensive, but not clear-cut. An official memorandum of the US Congressional Budget Office (CBO 1998) compared the findings of fourteen empirical studies regarding the effect of public pension provision on individual savings in the US. Two of these studies, David and Menchick (1985) and Gullason et al. (1993), find a positive effect (with elasticity between 0.1 and 0.5), but another five find elasticities ranging between -1 (the value predicted by life-cycle theory) and -0.2. The remaining seven find that the effect is either statistically insignificant (at the 5% level) or very close to zero.

In comparing micro with macro-data evidence, however, it must be kept in mind that the family constitution hypothesis implies a negative effect of the average pension contribution rate on the aggregate fertility rate, and a positive one on the aggregate savings rate, essentially because an increase in the contribution rate induces a number of adults, who would otherwise have children and save little, to have no children and top-up their pension entitlements with extra savings.\(^{49}\) There is thus an implied assumption that people can provide for old age by lending in the credit market, or buying assets. Consistently with this, Cigno and Rosati (1992) estimate that financial market development reduces the fertility rate and raises the saving rate, and Galasso et al. (2009) find that the fertility effect of public pension coverage is more strongly negative in the presence of well developed financial markets.

If it is true that pension coverage reduces aggregate fertility only or primarily because it reduces the share of compliers in the adult population, this effect will be zero if the share of compliers is already zero. This brings up the question, how likely is it that social security will entirely crowd-out family-based arrangements? Not very likely if neither the social services nor the market offer perfect substitutes for the personal care and attention that adults give to their infant children and elderly.

\(^{49}\) That is strictly true in the basic version of the model, where go-it-aloners have no children at all. In the extended version with descending altruism, a person may have children with either strategy, but will have fewer if she decides to go it alone than if she decides to comply.
parents. In the absence of systematic sampling over a long period of
time, we cannot say whether the amount of old-age care provided by
family members has declined, and by how much, in response to public
pension coverage: There is evidence, however, for example in Cigno and
Rosati (2000) and Lundberg and Pollak (2007), that the amount of care
the elderly receive from their relatives is still substantial, and may be
gaining in importance with the increase in the probability of living to a
very old age and thus of becoming disabled.

5 Optimal policy

We have looked at the consequences of public pension provision, but
we have not yet posed the question whether pension systems (more
generally, policy towards the old) as we know them are optimally de-
dsigned. The answer depends on expectations, the structure of informa-
tion and the underlying behavioural model. Assuming rational expec-
tations, symmetric information and no free-riding, we saw in Section
3 that, in the absence of other policies, the introduction of a conven-
tional pension system would raise welfare in the long run if individuals
behave in the way described by either the augmented life-cycle or the
descending-altruism model, raise welfare or leave it unchanged if their
behaviour is better described by the ascending-altruism model, reduce
welfare if the constitution model is the one closest to the truth. If we
drop rational expectations or allow for free-riding, however, the policy
will give rise to a negative fiscal externality, because individuals will not
take into account the effects of their fertility and human capital invest-
ment decisions on the pension system’s balance sheet. Consequently,
their children’s collective capacity to pay pension contributions when
they become adults will be inefficiently low.

Using an augmented life-cycle model, Groezen et al. (2003) show
that introducing a subsidy increasing in the number of children ("child
benefits") and financed by a lump-sum tax on adults alongside an un-
funded pension scheme would generate the right fertility incentives and
implement a social optimum. Analogous results were obtained by Peters
(1995), and Kolmar (1997). That is so, however, only because, in that
model, infant consumption is a given constant. If we allow \( c_0 \) to be cho-

n by the parent, child benefits will trigger a substitution of \( n \) for \( c_0 \),
and the resulting allocation will be inefficient. Furthermore, if parents
expend resources on their children’s education as well as consumption,
and education enhances a child’s future earnings (hence, pension con-
tributions), child benefits will raise the number of future contributors, but
reduce the amount each of them will contribute. Depending on relative
elasticities, therefore, the policy may either enhance or imperil the pen-
sion system’s financial viability. A first-best social optimum will come about only if the government has perfect control over education.

In reality, however, education does not end at school, and much of what goes on outside school is private information. Besides, as the child’s future earning capacity is a random variable with probability density conditional not only on education received, but also on the child’s innate talent and on how hard she will first study and then work, it is not possible to infer educational investment from labour market outcomes. A similar problem arises with respect to fertility. If parents could deterministically choose how many children to have (as in the simple models examined so far), and given that this number is common knowledge, the government could make subsidies to parents conditional on number of children as suggested in Groezen et al. (2003). In reality, however, the number of births is a random variable with probability density conditional on the parent’s reproductive behaviour and personal characteristics, both of which are private information. Given these moral hazard and adverse selection problems, a first best is out of reach.

The design of the second-best policy has the structure of a principal-agent problem with the government in the role of principal and would-be parents in that of agents.\textsuperscript{50} Cigno et al. (2003b) and Cigno and Luporini (2011) show that this policy taxes each adult in accordance with her earnings, and subsidizes her in accordance with her children’s aggregate earning capacity.\textsuperscript{51} This provides each adult with an incentive to have children and invest in their education. Given that a person’s earning capacity becomes apparent only when she is in middle life, and her parent on the point of retirement, the subsidy is best interpreted as a pension entitlement conditional on the quantity and quality of her children, and the tax paid as a pension contribution. The argument is thus similar to the one used by Cremer and Pestieau (1996) to reach the conclusion that parents should wait as long as possible before giving their children money (see Subsection 2.2).

Given that the family transfers money and personal services from adults to infants and the old, the question that now comes up is whether,\textsuperscript{50} For a survey, see Cigno (2011).
\textsuperscript{51} The first of these papers assumes that an adult can deterministically choose how many children to have. Given that the number of children is common knowledge, the policy maker will then use a forcing contract to fix this variable, and use distortionary incentives only to encourage imperfectly observable parental investments in education. In the second paper, the number of births is a random variable with probability density conditional on parental reproductive behaviour, and the second-best subsidy is then an increasing function of the children’s total earning capacity. In either model, the benefit schedule is non-linear because of the tension between incentive and insurance considerations.
or to which an extent, the government should replace the family in this role. As first pointed out in Ben-Porath (1980), a family may be too small, and its members too much alike, to permit efficient specialization and risk sharing. On the other hand, however, the family has an informational advantage over the government, because its members know each other’s characteristics better, and are also better placed to monitor each other’s behaviour, than any government official could. There is thus a trade-off between scale and informational considerations. Following this line of thought, Cigno (2010) proposes a pension system consisting of two parallel schemes, (a) a conventional Bismarck-type scheme allowing an adult to qualify for pension benefits by working and paying pension contributions in the usual way, and (b) an unconventional scheme allowing her to qualify for a pension benefit by raising children, and investing in their future earning capacity. In a second-best perspective, the benefit formulae would be a compromise between incentive and insurance considerations. Individuals should be free to mix, or switch in and out of, the two schemes at will, and thus to specialize to the extent they think fit in either child-raising or money-raising activities.

Finally, there is the question whether the government should limit itself to taxing and subsidizing, or provide services directly to the old. Pestieau and Sato (2006) enlarge the government’s armoury by introducing public nursing homes and subsidies for adults who care for disabled parents (as an alternative to placing them in a nursing home). The only source of heterogeneity in this model are the children’s wage rates, which determine both their ability to make cash transfers to their parents, and their opportunity-cost of personally caring for the latter. The underlying behavioural model is characterized by ascending altruism like those examined in Subsection 2.3. Within the logic of ascending altruism, a parent will strategically make transfers to her children to induce them to help her with cash or personal care if she becomes disabled. When she makes those transfers, the parent knows her children’s wage rates, but does not know whether she will become disabled. The government weighs the costs of providing public nursing homes or subsidizing children to provide care for their parents. In the second-best solution, the quality of the public nursing homes and the level of the public subsidy depend on the wage-rate distribution, and on the effects of the two policy instruments on private cash and time transfers. As usual, low-wage children will respond to the policy by personally caring for their disabled parents, and high-wage ones by assisting them with cash transfers.

52 Insurance against the risk a low realization of the pensioner’s own earnings in scheme (a), of a low realization of her children’s earning capacity in scheme (b).
6 Conclusion

The economic literature has moved a long way from the paradigm of the solitary individual borrowing and lending in a perfect capital market. As we saw in Section 2, we only have to recognize that the market does not lend to infants for the paradigm to fall apart. That particular problem is remedied by assuming altruism in some form, but the life-cycle allocation of consumption will still be inefficient. Problems of a different nature arise if we recognize that the elderly derive utility not only from market goods, but also from their children’s personal services, and these services do not have a perfect market substitute. If parents are altruistic towards their children, but not (or not as much) the other way round, the problem is that the children can coalesce against their parent and extract a monopoly price for their services. If children are altruistic towards their parent, but not (or not as much) the other way round, the problem is free-riding.

We saw in Section 4 that a possible remedy against opportunistic behaviour on the part of family members is a self-enforcing, renegotiation-proof, family constitution akin to the political constitution that prevents opportunistic behaviour on the part of successive parliaments. That conclusion was reached, however, using a highly stylized model where reproduction occurs by parthenogenesis. The moment we introduce sexual reproduction, the actor becomes the couple rather than the individual, and we run up against the problem that each couple has to contend with two family constitutions. Further complications arise if we allow for divorce and remarriage. The formal analysis of realistic family situations is still in its infancy.

Government provision of pensions and social services for the elderly widens the range of opportunities open to individuals in one direction, but restricts it in another because, as we saw in sections 3 and 4, the policy will tend to discourage altruistically motivated support for the elderly on the part of their grown-up children or able-bodied partners, and to crowd out self-enforcing family-level arrangements. We argued, however, that total crowding-out is unlikely, because the family supplies services for which neither the market nor the public sector has a perfect substitute, and that the family has informational advantages over those larger and impersonal organizations. As shown in Section 5, the second-best policy trades-off the advantages of family-level arrangements against the risk-sharing and risk-pooling opportunities offered by

\[53\] One instance of such behaviour is the accumulation of public debt, that will have to paid by future generations. A constitutional clause prescribing that the government must break even every year, or every legislature, will stop that.
the public sector and the market..

7 References


Bernheim, B. D. and D. Ray (1989), Collective Dynamic Consistency in Repeated Games, *Games and Economic Behavior* 1, 295-326


——— (2010), How to Avoid a Pension Crisis: A Question of Intelligent System Design, CESifo Economic Studies 56, 21-37


———, L. Casolaro and F. C. Rosati (2003a), The Impact of Social Security on Saving and Fertility in Germany, FinanzArchiv 59, 189-211


———, A. Luporini and A. Pettini (2003b), Transfers to Families with Children as a Principal-Agent Problem, Journal of Public Economics 87, 1165-1172


——— and F. C. Rosati (1992), The Effects of Financial Markets


Hohm, C. H. (1975), Social Security and Fertility: An International Perspective”*, *Demography* 12, 629-644


Maskin, E. and J. Farrell (1989), Renegotiation in Repeated Games, *Games and Economic Behavior* 1, 327-360

nomics 26, 1233-1250


Pestieau P. and M. Sato (2006), Long Term Care: the State and the Family, Annales d‘Economie et de Statistique 83-84, 151-166


Stark, O. (1993), Nonmarket Transfers and Altruism, European Economic Review 37, 1413-1424


Economics 12, 625-640

